Effects of Turbulence on the Femtosecond pulses for Free-space Optical Transfer Link

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***Abstract*—With the continued improvements in quantum frequency standards, there has been a brisk demand for free-space optical links for time and frequency transfer. The effects of turbulence on the femtosecond pulses need to be further clarified. In this work, we explore the effects of turbulence on the comb pulses and dual-comb interference signals with theoretical analysis. A comb-based free-space optical transfer link is used for verification with the use of a reciprocal optical terminal. The results indicate that atmospheric turbulence does aggravate the relative linewidth and phase noise of the dual-comb interference signals.**

Keywords—femtosecond comb, turbulence, optical transfer link

I. INTRODUCTION

With the continued improvements in quantum frequency standards, there has been a brisk demand for free-space optical links for time and frequency transfer [1], towards the goal of future free-space networks for applications ranging from global positioning and timing to fundamental science.

To cancel out the variations in the time-of-flight of the optical links across turbulent air, the reciprocity of the optical bidirectional is used as a dominant approach. Recent free-space optical two-way time-frequency transfer based on optical frequency combs has verified the levels of 100 as in time rely on this reciprocity [2,3]. However, in future realizations of satellite-to-ground or inter-satellite networks, these links will extend over long distances and will cover the moving platforms. In these applications, the spatial variations of the atmospheric turbulence will destroy the reciprocity, and the effects of turbulence on the comb pulses need to be further clarified.

In this paper, we explore the effects of turbulence on the comb pulses and dual-comb interference signals with theoretical analysis. A comb-based free-space optical transfer link is used for verifying the analysis with the use of a reciprocal optical terminal. The asynchronous sampling process maps the effects to the changes in the dual-comb interference signals, and the experimental results indicate that turbulence does aggravate the relative linewidth and phase noise of the dual-comb interference signals.

II. THEORY

A. Turbulence-induced jitter for femtosecond pulses

In the time domain, the nth pulse of a femtosecond comb can be expressed as:

$$e_{\mathrm{in},\,n}(t)=A\left(t-\frac{n}{f_{\mathrm{r}}}\right)\exp\left(i\omega_0 t+i2\pi n\frac{f_{\mathrm{ceo}}}{f_{\mathrm{r}}}\right),\quad (1)$$

where A is the electric field envelope of the pulse and is a function of time t, f_{r} is the repetition frequency, f_{ceo} is the carrier envelope offset, and ω_0 is the central frequency of the laser.

In the frequency domain, the nth pulse can be expressed with Fourier transform:

$$E_{\mathrm{in},\,n}(\omega)=\mathrm{F}\left\{e_{\mathrm{in},\,n}(t)\right\}.\quad (2)$$

Here, the absorption of the atmospheric turbulence is assumed to be zero, since the discussion is focused on the effect of phase disturbance introduced by the turbulence on the pulses. The phase shift of the nth pulse after passing through the optical transfer link can be decomposed as ψ_{ideal}, (ideal phase shift) and $\psi_{\mathrm{tur},\,n}$ (turbulence-induced phase shift) [4]:

$$\begin{aligned}\psi_n(\omega)\approx\psi_{\mathrm{ideal}}(\omega_0)+\frac{\partial\psi_{\mathrm{ideal}}}{\partial\omega}(\omega-\omega_0)+\\ \psi_{\mathrm{tur},\,n}(\omega_0)+\frac{\partial\psi_{\mathrm{tur},\,n}}{\partial\omega}(\omega-\omega_0)\end{aligned}.\quad (3)$$

Higher order terms for dispersive effect are not considered here. Therefore, the above Taylor expansions of both the ideal and turbulence-induced phase shifts retain only the first two terms. What calls for special attention is that the ideal phase shift ψ_{ideal} remains constant for each pulse, while the turbulence-induced phase shift $\psi_{\mathrm{tur},\,n}$ is different from pulse to pulse, as is indicated by the subscript n.

After passing through a free-space optical transfer link, the nth pulse (represented by “out”) can be described as:

$$\begin{aligned}e_{\mathrm{out},\,n}(t)&=\mathrm{F}^{-1}\left\{E_{\mathrm{in},\,n}(\omega)\exp\left[i\psi_n(\omega)\right]\right\}\\&=\exp\left\{i\left[\psi_{\mathrm{ideal}}(\omega_0)-\frac{\partial\psi_{\mathrm{ideal}}}{\partial\omega}\omega_0+\psi_{\mathrm{tur},\,n}(\omega_0)-\frac{\partial\psi_{\mathrm{tur},\,n}}{\partial\omega}\omega_0\right]\right\}*\\&\quad\mathrm{F}^{-1}\left\{E_{\mathrm{in},\,n}(\omega)\left[\exp\left(i\omega\left(\frac{\partial\psi_{\mathrm{ideal}}}{\partial\omega}+\frac{\partial\psi_{\mathrm{tur},\,n}}{\partial\omega}\right)\right)\right]\right\}\end{aligned}\quad (4)$$

By combining (1) and using the property of Fourier transform, it can be finally obtained:

$$\begin{aligned}e_{\mathrm{out},\,n}(t)=\exp\left\{i\left[\omega_0 t+2\pi n\frac{f_{\mathrm{ceo}}}{f_{\mathrm{r}}}+\psi_{\mathrm{ideal}}(\omega_0)+\psi_{\mathrm{tur},\,n}(\omega_0)\right]\right\}*\\ A\left(t+\frac{\partial\psi_{\mathrm{ideal}}}{\partial\omega}+\frac{\partial\psi_{\mathrm{tur},\,n}}{\partial\omega}-\frac{n}{f_{\mathrm{r}}}\right)\end{aligned}.\quad (5)$$

According to the electric field expression (5), the ideal phase shift of the free-space optical transfer link introduces to the nth pulse a phase shift $\psi_{\mathrm{ideal}}(\omega_0)$ and a time shift $-\frac{\partial \psi_{\mathrm{ideal}}}{\partial \omega}$, which is the effect of the passage of an ideal optical link. Similarly, the turbulent one introduces a phase shift $\psi_{\mathrm{tur},n}(\omega_0)$ and a time shift $-\frac{\partial \psi_{\mathrm{tur},n}}{\partial \omega}$ for different pulses (n = 1, 2, 3…), which can be viewed as random variables.

To further quantify the turbulence-induced jitter of the carrier phase and time of flight from pulse to pulse, we can further define $\delta f_{\mathrm{r},n}$ and $\delta f_{\mathrm{ceo},n}$ such that:

$$-\frac{n}{f_{\mathrm{r}}+\delta f_{\mathrm{r},n}}=\frac{\partial \psi_{\mathrm{tur},n}}{\partial \omega}-\frac{n}{f_{\mathrm{r}}}, \tag{6}$$

$$2\pi n\frac{f_{\mathrm{ceo}}+\delta f_{\mathrm{ceo},n}}{f_{\mathrm{r}}+\delta f_{\mathrm{r},n}}=2\pi n\frac{f_{\mathrm{ceo}}}{f_{\mathrm{r}}}+\psi_{\mathrm{tur},n}(\omega_0), \tag{7}$$

which explicitly express turbulent phase shift in terms of the jitters of f_{r} and f_{ceo} for the femtosecond pulses:

$$\delta f_{\mathrm{r},n}=\frac{n}{-\frac{\partial \psi_{\mathrm{tur},n}}{\partial \omega}+\frac{n}{f_{\mathrm{r}}}}-f_{\mathrm{r}}. \tag{8}$$

$$\delta f_{\mathrm{ceo},n}=\left(f_{\mathrm{r}}+\delta f_{\mathrm{r},n}\right)\frac{\psi_{\mathrm{tur},n}(\omega_0)}{2\pi n}+\delta f_{\mathrm{r},n}\frac{f_{\mathrm{ceo}}}{f_{\mathrm{r}}}. \tag{9}$$

B. *Turbulence-induced jitter for dual-comb interferogram*

According to (5), the arrival time of the nth pulse is

$$t_n=\frac{\partial \psi_{\mathrm{tur},n}}{\partial \omega}+\frac{n}{f_{\mathrm{r}}} \tag{10}$$

Note that the 0th pulse corresponds to the 0th interferogram burst, and define the time at this moment as t = 0. The number of pulses contained in two adjacent interferogram bursts is defined as $N=\frac{f_{\mathrm{r}}}{\Delta f_{\mathrm{rep}}}$. Then the Nth pulse corresponds to the center of the first interferogram and the time should be expressed as:

$$t_N=\frac{\partial \psi_{\mathrm{tur},N}}{\partial \omega}+\frac{N}{f_{\mathrm{r}}} \tag{11}$$

Similarly, the time corresponding to the burst of the kth interferogram can be expressed as:

$$t_{kN}=\frac{\partial \psi_{\mathrm{tur},kN}}{\partial \omega}+\frac{kN}{f_{\mathrm{r}}} \tag{12}$$

which can be used to quantify the deviation of the center time of the kth interferogram from the theoretical value as:

$$\frac{\partial \psi_{\mathrm{tur},kN}}{\partial \omega}=t_{kN}-\frac{kN}{f_{\mathrm{r}}}=t_{kN}-\frac{k}{\Delta f_{\mathrm{rep}}} \tag{13}$$

III. RESULTS

To visualize the effects of turbulence on the comb pulses, we map the effects to the changes in the interference signals by the dual-comb asynchronous sampling process. A comb-based free-space optical transfer link is used with a reciprocal optical terminal [5]. The atmosphere is under well-mixed, the wind speed is ~1–3 m/s, and the path length is ~900 m. The relative linewidth and phase noise of the dual-comb interference signals before and after passing through the optical transfer link are compared in Fig. 1.

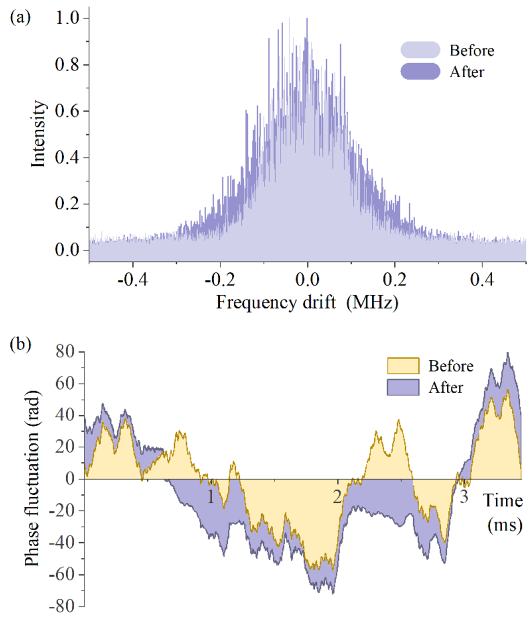

Fig. 1. Verification of the turbulence-induced influence on the femtosecond pulses of the comb. Comparison of the results before and after passing through the free-space optical transfer link. (a) Relative linewidth of the dual-comb teeth; (b) phase fluctuation of the dual-comb interference signal.

As shown in Fig. 1(a), the relative linewidth of one pair of comb-teeth is further broadened due to the turbulent phase noise. The deviation from the theoretical linear value of the phase of the dual-comb interference signal is shown in Fig. 1(b). The phase fluctuates randomly due to the source-induced time jitter, and turbulence further aggravates this phase fluctuation, which results in more serious distortions of the dual-comb interferogram.

IV. CONCLUSION

The effects of turbulence-induced phase noise can be divided into variations in zero-order, first-order, and higher-order terms of its derivative with respect to frequency. Turbulence-induced jitter of the carrier phase and time of flight for the femtosecond pulses can be quantified through the variation in the first-order term, and turbulence-induced jitter of the center time of the kth dual-comb interferogram also can be quantified in a similar form.

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